Preface

Twenty years ago, the World Scientific Publishers started a new review volume series entitled *Order, disorder and criticality. Advanced problems of phase transition theory.*[1] The original goal of this series was to demonstrate that the theory of phase transitions and critical phenomena, whose golden age was in the last decades of the 20th century, did not recede into the past along with that century, but has continued to develop and that there is still much to be said there, both at the fundamental level and in terms of applications. This initiative turned out to be successful, and evidence of this, in particular, is the eighth volume of the series, which is being released this year.

Over time, the subject matter of the series has gone beyond discussing phase transitions and critical phenomena in condensed matter. The scope of the reviews have included analysis of tipping points and emergence phenomena in biological, social, technological systems of many interacting agents. Of particular interest were cases when the behavior of such systems as a whole did not follow trivially from the behavior of their components. In this way, the subject matter of the series (the contents of the previous volumes are given on pp. x – xiii) reflects the dynamics of the development of the complex system science. The latter increasingly uses the methods and concepts of physics, notably of statistical physics and the physics of phase transitions. Therefore, our review series has been designed not only to reflect this trend, but also to contribute to its success. Starting with this volume, we explicitly mention complex systems in the series subtitle, underscoring this fact.

As the previous volumes, the current one addresses traditional physical problems, as well as those where a physical approach is extended to the

[1] *Order, Disorder and Criticality. Advanced Problems of Phase Transition Theory*, edited by Yu. Holovatch (World Scientific, Singapore), vol. 1 – 2004, vol. 2 – 2007, vol. 3 – 2012, vol. 4 – 2015, vol. 5 – 2018, vol. 6 – 2020, vol. 7 – 2023.

beyond-physics field. It starts with the chapter *The enigmatic exponent ϙ and the story of finite-size scaling above the upper critical dimension* by Ralph Kenna and Bertrand Berche. Sadly, Ralph Kenna,[2] passed away last year, and the chapter has been finalized by Bertrand Berche, paying tribute to their long-standing collaboration and friendship. The chapter gives a rather personal story about the evolution of concepts of scaling, hyperscaling and finite-size scaling in high dimensions. The authors belong to those who shaped our modern understanding of these concepts; therefore, such a story is of special value. Their review describes the main steps taken to understand criticality above the upper critical dimension. The fundamental idea implemented by the finite-size scaling concept is that there are only two relevant length scales for a system of finite size: system length L and correlation length ξ. Near the critical point, it is assumed that the correlation length becomes equivalent to the entire size of the system. This, in turn, leads to the finite-size scaling relations. A bold and, at first glance, unusual statement was to relax the assumption that the correlation length has to be bounded by the physical length. It has been suggested that its growth with L is governed by the new critical exponent ϙ: $\xi \sim L^{ϙ}$. The growth is linear, below the upper critical dimension; however, it becomes superlinear at high dimensions, allowing to extend the validity of hyperscaling and finite-size scaling to the high-dimensional regime.

The perspective of phase transitions, scaling, and universality is also important for understanding the next chapter of this book - *Stochastic spatial Lotka–Volterra predator-prey models* by Uwe Täuber. This chapter discusses how methods of statistical physics in analytical and computational approaches allow an insight into and quantitative description of spontaneous pattern formation and other noise-induced phenomena occurring in the interacting populations. To this end, recent studies of the generalizations of the famous Lotka-Volterra predator-prey model[3] are considered. In the original setting, the dynamics of the predator-prey interplay has been described via a system of coupled mean-field rate equations. Yet another perspective opens, when one modifies the model to include stochasticity and spatial extension. This strategy has been successfully implemented by

[2] Ralph Kenna (August 27, 1964 – October 26, 2023) was an Irish scholar in the field of statistical physics and complex system science. See more about him in the special issues of the journals *Condensed Matter Physics* **27** (2024) and *Entropy* **26** (2024).

[3] As I write this preface in Lviv, Ukraine, I can't help but mention that Alfred J. Lotka was born here, and that the Ukrainian chemist Julian Hirniak, another Lviv native quoted by Lotka, was the proponent of periodic chemical reactions as early as in 1908. See more in N. Manz, Yu. Holovatch, J. Tyson, to appear (2024).

the author of this chapter and his colleagues, having in mind applications in various fields ranging from physics and chemistry to biology, ecology, epidemiology, and even sociology. Both the models considered and the methods used in their analysis enable concentrating on the fluctuation and correlation effects that are not captured by the mean-field treatment. This, in turn, leads to subtle effects that can be interpreted as the far-from-equilibrium continuous phase transitions within the directed percolation universality class.

The third chapter of the book, *Statistical Mechanics and Artificial Neural Networks: Principles, Models, and Applications* by Lucas Böttcher and Gregory Wheeler, consists of two major parts. The first part can be read independently and serve as a short but self-consistent and self-contained introduction to the statistical mechanics of artificial neural networks (ANNs). As the authors themselves note, contributions to ANN research extend beyond statistical mechanics: "...Historically, these efforts have been marked by eras of interdisciplinary collaboration referred to as cybernetics, connectionism, artificial intelligence, and machine learning".[4] However, the role that statistical physics played and continues to play in these studies is difficult to overestimate, as the overview presented in this chapter aims to emphasize. Actually, some versions of artificial neural networks were inspired by the Ising model. This connection is explained in detail, taking the Hopfield model and Boltzmann machines as the examples and providing an overview of the other models, principles, and applications of ANNs. Such interpretation of concepts from the language of learning systems to that of statistical physics is of particular value for the uninitiated reader. The second part of this chapter discusses geometric properties of ANNs. Since the latter can be seen as high-dimensional mathematical functions, analysis of their landscapes in the high-dimensional space (the so-called loss landscapes) serves as an efficient tool to acquire information about their performance.

Whereas an approach to and a depart from a certain state is of equivalent relevance and is studied by using basically similar tools in settings of theoretical physics, this is not the case for a social system. As it is emphasized in the chapter *Political systems as complex, statistical systems – Unravelling the role of diversity and feedback for stability* by Karoline Wiesner, much research has focused on how democracies arise and can be facilitated; however, much less is known about the destabilization of

[4]See p. 118 of this book.

democracies. The latter phenomenon is termed in the political sciences as democratic backsliding. The chapter summarizes recent studies relating to these questions and shows how the complexity science and statistical mechanics are becoming useful tools in creating modern theories of political instability. Like the former chapter, this one also comprises two parts. The first part discusses inherent features of complex systems, relying on the taxonomy introduced in a recent book written in a cooperation with the current author.[5] Further discussion in the second part of the chapter focuses on three features of complex social systems, showing that their mutual relationship is of particular relevance for understanding the rise and fall of democracies. These are disorder, feedback, and stability. Qualitative conclusions of the discussion are supported by a recent quantitative study of data provided by the Varieties of Democracy project.

Concepts of scaling and universality are further discussed in the last chapter of the book, *Observing cities as a complex system* by Rafael Prieto-Curiel. Here, the quest for universality is exemplified by analyzing data about different cities. Specific constraints (history, geography, economy) are very important locally, but are there features of large cities that have some universal character? If so, what are these features and what do they indicate? The search for answers to these and similar questions has led to what is now called the science of cities, and the consideration of a city as a complex system is an important component of such research. In particular, the urban scaling theory gives a framework for analysing cities in the context of their size. The chapter reviews some urban scaling principles as well as approaches for analysing variations between different indicators for areas within the same city. In addition, the chapter provides an overview of the acceleration of the dynamics of global urbanization and provides numerous examples of studies of cities as complex systems.

This review series originates from the Ising Lectures,[6] an annual Workshop on critical phenomena and complex systems that started in Lviv in 1996. Year 2024 marks the centennial anniversary of Ernst Ising's doctoral thesis[7] defence and, with our book, we also mark this anniversary. I am

[5] J. Ladyman, K. Wiesner, *What Is a Complex System?* Yale University Press, New Haven & London, 2020.

[6] http://www.icmp.lviv.ua/ising/

[7] The now famous Ising model was described and solved in 1D in Ernst Ising's doctoral thesis written under supervisorship of Wilhelm Lenz: *Beitrag zur Theorie des Ferro- und Paramagnetismus. Dissertation zur Erlangung der Doktorwürde der Mathematisch–Naturwissenschaftlichen Fakultät der Hamburgischen Universität vorgelegt von Ernst Ising aus Bochum. Hamburg 1924.*

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